## ACTIVITY TWO CONSTANT VELOCITY IN TWO DIRECTIONS

## Purpose

The overall goal of this activity is for students to analyze the motion of an object moving with constant velocity along a diagonal line. In this experiment, the Motion Visualizer (MV) captures the path of a red buggy moving diagonally across the camera field of view. The MV assists in the resolution of displacement and velocity in x and y components. The concept of displacement and velocity as vector quantities are studied.
After this activity, students should be able to do the following:
$\sqrt{ }$ Describe, in words, what is meant by an object moving in two dimensions.
$\sqrt{ }$ Predict how this motion will be describe on the following types of graphs: Distance v. Time, Position v. Time, Speed v. Time and Velocity v. Time.
$\sqrt{ }$ Define the following words: scalar, vector, distance, displacement, velocity, speed, component and resultant.
$\sqrt{ }$ Compute a vector component: either a displacement or velocity component.
$\sqrt{ }$ Compute the resultant vector using trigonometric functions.
can happen in more than one dimension.

## Materials

- An object that can move with constant velocity.
[Recommendation: Red Pasco Constant Velocity Buggy]
- Computer with Motion Visualizer software
- Digital video camera with tripod
- Masking tape and marker
- Angle measuring device
- Tape measure or meter stick



## Set-up Guidelines

1. Make a Cartesian coordinate system with two pieces of 2-meter long masking tape on a smooth 3-meter by 3-meter surface.
2. Label the two pieces of tape as the $X$ and $Y$-axes and mark the origin.
3. Mark out a 1.5 to 2 meter track on the surface.
4. Put pieces of tape on the surface to mark the start and end of the track.
5. Place front end of buggy at start.
6. While one person is holding the buggy, turn it on.
7. Another person should start the video camera [in record mode - no tape needed.]
8. Let buggy roll.
9. Capture the motion of the buggy with digital video camera.

## Diagram

[Note: Refer to the MV Quick Start Guide for a complete description of equipment set-up. In the guide, you will find a comprehensive description of how the video camera should be set-up and the settings needed for each type of activity. You will also find instructions on how the Motion Visualizer software can be used to capture the motion of the object.

Below, you will find a diagram of how the equipment was used for this specific activity. This apparatus design works well but is just one of many possibilities. We encourage you to design your own set-up based on the materials available.]

## Side View of Experimental Set-up



## Bird's Eye View of Experimental Set-up



Axes Lengths $=2.0 \mathrm{~m}$

In this activity, the buggy moved from quadrant III to quadrant I of the Cartesian coordinate system, passing very close to the origin. This motion shows up when viewed in the Room Coordinate Graph. Any path could be taken and we encourage you to try different various directions to compare and contrast the data.

## Data Collection and Presentation Guidelines

There are many ways to use the MV software to display the motion of an object; however, it is important to keep in mind the objective of the activity. This will help prevent the user of the software from displaying redundant or unnecessary information.
In this activity, we were interested in studying the motion of an object moving with a constant velocity in the X and Y planes. Knowing this, we consciously did not display data that seemed irrelevant [e.g., Z Position v. Time, Z Velocity v. Time, etc.] Rather, we deliberately chose the graphs that would best demonstrate constant velocity motion in two dimensions. The graphs and analyses included in this activity description are here to provide you with ideas on how you can use this data with your students.
The following is a suggested listing of the graphs, with analyses, that can be used with this activity.

1. Room Coordinate Graph - This visual shows the path that the object took in the X and $Y$ directions. Since the motion happened in only the X and Y directions, it is recommended that this graph include only the X and Y axes and that the Orthographic View is chosen. It may be of interest for students to see the motion from different perspectives
2. $\underline{X}$ Position v. Time Graph - This is a graphical interpretation of the object's displacement in the X direction.

- Students can describe, with words, the motion of the object as depicted by this graph.
- The total distance traveled in this direction can be determined.
- A discussion about vectors can be included here.
- The total distance traveled in this direction is just one component of the total distance traveled.
- Students can draw a best-fit line and calculate the slope of this line.
- The meaning of the slope of a line on this type of graph can be discussed.
- The value can then be compared with the X Velocity v. Time Graph.

3. Y Position v. Time Graph - This is a graphical interpretation of the object's displacement in the Y direction.

- Students can describe, with words, the motion of the object as depicted by this graph.
- The total distance traveled in this direction can be determined.
- A discussion about vectors can be included here.
- The total distance traveled in this direction is just one component of the total distance traveled.
- Students can draw a best-fit line and calculate the slope of this line.
- The meaning of the slope of a line on this type of graph can be discussed.
- The value can then be compared with the X Velocity v. Time Graph.
- Students can combine the two displacement vectors and arrive at a resultant displacement vector.
- The numerical value of this result can be compared to the actual distance traveled by the object on the floor.

4. $\underline{X}$ Velocity v . Time Graph - This is a graphical interpretation of the object's velocity in the X direction.

- Students can describe, with words, the motion of the object as depicted by this graph.
- Students can draw a best-fit line and calculate its slope.
- The meaning of the slope of a line on this type of graph can be discussed.
- A discussion of what this graph’s slope means for the motion can be addressed.
- The idea that this is one component of the object's velocity can be discussed.

5. Y Velocity v. Time Graph - This is a graphical interpretation of the object's velocity in the Y direction.

- Students can describe, with words, the motion of the object as depicted by this graph.
- Students can draw a best-fit line and calculate its slope.
- The meaning of the slope of a line on this type of graph can be discussed.
- A discussion of what this graph's slope means for the motion can be addressed.
- The idea that this is one component of the object's velocity can be discussed.
- Students can combine the two velocity vectors and arrive at a resultant velocity vector. The numerical value of this result can be compared to the average speed of the vehicle as shown on the Speed v. Time graph.

6. Speed v. Time Graph - This is a graphical interpretation of the object's combined $X$ and $Y$ speeds.

- Students can describe, with words, what this graph tells them.
- Students can draw a best-fit line and calculate its slope.
- The meaning of the slope of a line on this type of graph can be discussed.
- A discussion of what this graph's slope means for the motion can be addressed.
- Students can determine the average speed of the object. The idea that this value represents the numerical portion of the resultant velocity can be discussed.


## Data Analysis

This is the image of the Room Coordinate Graph for this activity. Since this motion happens in only the X and Y directions, it is recommended that the Orthographic View be chosen and that the graph only include the X and Y axes.


This is an image of the X Position v. Time graph for this activity. Since the car moved at a constant velocity from a position on the left of the origin to a position to the right of the origin, the X Position v. Time graph has a non-curved line with a positive slope, indicating a positive direction of travel. Notice the object started at about -0.50 meters and ended at approximately +0.75 meters. The total distance traveled in the X direction was therefore 1.25 meters.


The basic X Position v. Y Position graph clearly shows that the car is moving in both the x and y direction. It is easy to estimate the angle the car is traveling in relationship to both the x and y axis to be approximately $45^{\circ}$. This can also be done by resolving the displacement vector into its $x$ and $y$ components. Then $\theta$ can be found by taking $\tan ^{-1}$ ( $y$ displacement/ $x$ displacement)


The correlation between this graph and the separate X Position v. Time and Y Position v. Time graphs can be shown by calculating the same angle this time by finding the displacement in both planes on the X and Y Position v. Time graphs.



From the calculations of x and y displacement it follows that:
$\theta=\tan ^{-1}(1.2 \mathrm{~m} / 1.3 \mathrm{~m})=43^{\circ}$
(This is a little more accurate than just estimating the angle as $45^{\circ}$ )
After $\theta$ is found it is easy to find what the velocity should be using information supplied in the Speed v. Time graph and some more basic trig.


## We find that:

$\mathrm{V}_{\mathrm{x}} \approx .5\left(\cos 43^{\circ}\right) \approx .36 \mathrm{~m} / \mathrm{s}$
$\mathrm{V}_{\mathrm{y}} \approx .5\left(\sin 43^{\circ}\right) \approx .34 \mathrm{~m} / \mathrm{s}$
Written by Michael Quinlan. Michael is a Physics teacher at Newton North High School, Newton, Massachusetts

In other words the velocity in the $x$ direction should be slightly higher than the velocity in the $y$ direction. We can easily check to see that this corresponds with the data obtained from the X and Y Velocity v. Time graphs.


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## Questions for Discussion

## Review

1. How would you describe the shape of the X and Y Position v. Time graphs?
2. What does the slope of the $X$ and $Y$ Position v. Time graphs represent? Check to see if the slope of these lines is equal to the values for x and y velocity given by the X and Y Velocity v. Time graphs. Why can you use the slope of this line to estimate the velocity?

## Basic Assessment

3. Use two different graphs to find the displacement in the x direction. Again, use two graphs to find the displacement in the y direction.
4. Can you find the speed of the object by algebraically adding the $x$ and $y$ components of the velocity? Why or why not? Try this out by estimating the value given for the velocity in both the x and y directions shown by MV and comparing their sum to the speed graph.
5. What mathematical model allows you to correctly compute speed of the car?
6. Why is it necessary to represent the motion of the car by using vectors? Was it necessary to represent the motion of the car as a vector when it is only moving in the $x$ direction (think of the first activity)?
7. If you change the path of the car to be at a different angle from the horizontal how does this change the graphs of $x(t), y(t), v x(t)$, and $v y(t)$ ?
8. Should changing the direction of the path of the car also affect the graph of the speed? Why or why not?
9. Why is it important to break up a vector, such as the motion of the car, into its component parts? How could this help you analyze the data shown? Can you think of how the concepts that you have learned that you could use to your advantage in every day life? Hint: think of a police car.
10. Why doesn't the velocity in both the x and y direction remain constant throughout the trip of the buggy?

## Bridge to Next Activity

11. What do you think would cause the velocity of the car to change in either direction?
12. Would it be possible to make the velocity change in one direction and not the other?

## Extensions

The red buggy was approximately situated at a 45 degree angle across the field of view. In the extension experiment adjust the angle from the horizontal at which the cart crosses the field of view. How will the vector components of displacement and velocity differ at 15, 30, 60, and 75 degrees?

The world of vectors and motion is all around us. Track the motion of a walking pedestrian crossing a street with no traffic, varying angle connecting two points on opposite sides of the street. What is the fastest way to cross the street given constant velocity? What is the optimal path for greatest displacement in a given time interval?

Use the MV to the collisions of two equal mass pucks on an air table. What do you notice about the magnitude and direction of the post-collision velocities to the pre-collision velocities?

